

Electromagnetic properties of neutrinos

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Abstract. A short review on electromagnetic properties of neutrinos is presented. In spite of many efforts in the theoretical and experimental studies of neutrino electromagnetic properties, they still remain one of the main puzzles related to neutrinos.

Neutrino electromagnetic vertex function. The neutrino electromagnetic properties (see [1] for a recent review on this topic and the corresponding references on original papers) are determined by the neutrino electromagnetic vertex function $\Lambda_\mu(q, l)$ that is related to the matrix element of the electromagnetic current between the neutrino initial state $\psi(p)$ and final state $\psi(p')$, $\langle \psi(p') | J_\mu^{EM} | \psi(p) \rangle = \bar{u}(p') \Lambda_\mu(q, l) u(p)$, where $q_\mu = p'_\mu - p_\mu$, $l_\mu = p'_\mu + p_\mu$. Lorentz and electromagnetic gauge invariance imply [1] (see also [2, 3, 4]) that the vertex function can be written in terms of four form factors,

$$\Lambda_\mu(q) = f_Q(q^2) \gamma_\mu + f_M(q^2) i \sigma_{\mu\nu} q^\nu + f_E(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 + f_A(q^2) (q^2 \gamma_\mu - q_\mu \not{q}) \gamma_5, \quad (1)$$

where $f_Q(q^2)$, $f_M(q^2)$, $f_E(q^2)$ and $f_A(q^2)$ are charge, dipole magnetic and electric, and anapole neutrino form factors. The matrix element of the electromagnetic current can be considered also between neutrino initial $\psi_i(p)$ and final $\psi_j(p')$ states with different masses, $p^2 = m_i^2$ and $p'^2 = m_j^2$. The corresponding vertex function can be written in the form

$$\Lambda_\mu(q) = \left(f_Q(q^2)_{ij} + f_A(q^2)_{ij} \gamma_5 \right) (q^2 \gamma_\mu - q_\mu \not{q}) + f_M(q^2)_{ij} i \sigma_{\mu\nu} q^\nu + f_E(q^2)_{ij} \sigma_{\mu\nu} q^\nu \gamma_5, \quad (2)$$

where the form factors are matrices in the space of neutrino mass eigenstates [5].

In the case of Dirac neutrinos, the assumption of CP invariance combined with the hermiticity of the electromagnetic current J_μ^{EM} implies that the electric dipole form factor vanishes. In the case of Majorana neutrinos, regardless of whether CP invariance is violated or not, the charge, dipole magnetic and electric form factors vanish [3, 6], $f_Q = f_M = f_E = 0$ (the anapole moment can be non-vanishing, see also [7], as well as transition magnetic and electric moments). Since Dirac and Majorana neutrinos exhibit quite different electromagnetic properties, the investigation of neutrino electromagnetic interactions provides a tool for specifying the neutrino nature.

Neutrino electric form factor. It is usually believed that the neutrino electric charge is zero. This is often thought to be attributed to gauge invariance and anomaly cancellation constraints imposed in the Standard Model. In the Standard Model of $SU(2)_L \times U(1)_Y$ electroweak interactions it is possible to get [8] a general proof that neutrinos are electrically neutral which is based on the requirement of electric charges quantization. The direct calculations of the neutrino charge in the Standard Model for massless (see, for instance [9] and references therein) and massive neutrino [10] also prove that, at least at the one-loop level, the neutrino electric charge is gauge independent and vanishes. However, if the neutrino has a mass, the statement that a neutrino electric charge is zero is not so evident as it meets the eye. As a result, neutrinos may become electrically millicharged particles [8].

The most severe experimental constraints on the electric charge of the neutrino, $q_\nu \leq 10^{-21}e$, are obtained assuming electric charge conservation in neutron beta decay $n \rightarrow p + e^- + \nu_e$, from the neutrality of matter (from the measurements of the total charge $q_p + q_e$) [11]. A detailed discussion of different constraints on the neutrino electric charge can be found in [12].

Even if the electric charge of a neutrino is vanishing, the electric form factor $f_Q(q^2)$ can still contain nontrivial information about neutrino static properties. A neutral particle can be characterized by a superposition of two charge distributions of opposite signs so that the particle's form factor $f_Q(q^2)$ can be non zero for $q^2 \neq 0$. The mean charge radius (in fact, it is the charged radius squared) of an electrically neutral neutrino is given by $\langle r_\nu^2 \rangle = -6 \frac{df_Q(q^2)}{dq^2} \big|_{q^2=0}$, which is determined by the second term in the expansion of the neutrino charge form factor $f_Q(q^2) = f_Q(0) + q^2 \frac{df_Q(q^2)}{dq^2} \big|_{q^2=0}$ in series of powers of q^2 .

Note that there is a long standing discussion (see [1] for details) in the literature on the possibility to obtain (calculate) for the neutrino charged radius a gauge independent and finite quantity. In the corresponding calculations, performed in the one-loop approximation including additional terms from the $\gamma - Z$ boson mixing and the box diagrams involving W and Z bosons, the following gauge-invariant result for the neutrino charge radius have been obtained [13]: $\langle r_{\nu_l}^2 \rangle = \frac{G_F}{4\sqrt{2}\pi^2} \left[3 - 2 \log \left(\frac{m_l^2}{m_W^2} \right) \right]$, where m_W and m_l are the W boson and lepton masses ($l = e, \mu, \tau$)¹. Numerically, for the electron neutrino electroweak radius it yields $\langle r_{\nu_e}^2 \rangle = 4 \times 10^{-33} \text{ cm}^2$. This theoretical result differs at most by one order of magnitude from the available experimental bounds on $\langle r_{\nu_l}^2 \rangle$. Therefore, one may expect that the experimental accuracy will soon reach the value needed to probe the neutrino effective charge radius.

Neutrino magnetic and electric moments. The neutrino dipole magnetic moment (along with the electric dipole moment) is the most well studied among neutrino electromagnetic properties. A Dirac neutrino may have non-zero diagonal electric moments in models where CP invariance is violated. For a Majorana neutrino the diagonal magnetic and electric moments are zero.

The explicit evaluation of the one-loop contributions to the Dirac neutrino dipole moments in the leading approximation over the small parameters $b_i = m_i^2/m_W^2$ (m_i are the neutrino masses, $i = 1, 2, 3$), that however exactly accounts for $a_l = m_l^2/m_W^2$, leads to the following result [15]:

$$\left. \begin{matrix} \mu_{ij}^D \\ \epsilon_{ij}^D \end{matrix} \right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2} \left(1 \pm \frac{m_j}{m_i} \right) \sum_l f(a_l) U_{lj} U_{li}^*, \quad f(a_l) = \frac{3}{4(1-a_l)^3} (2 - 7a_l + 6a_l^2 - a_l^3 - 2a_l^2 \ln a_l), \quad (3)$$

where U_{li} is the neutrino mixing matrix. From (3) in the limit $a_l \ll 1$, the diagonal magnetic moment of a Dirac neutrino is given by [5] $\mu_{ii}^D = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1 \text{ eV}} \right) \mu_B$. On the other hand, the magnetic moment of a hypothetical heavy neutrino ($m_\ell \ll m_W \ll m_\nu$) is given by [10] $\mu_\nu = \frac{eG_F m_\nu}{8\sqrt{2}\pi^2}$. Note that much larger values for the neutrino magnetic moments can be obtained in various extensions of the Standard Model (see, for instance, [1]).

¹ This result, however, revived the discussion [14] on the definition of the neutrino charge radius.

Bounds on neutrino magnetic moments. Constraints on the neutrino magnetic moment have been obtained in laboratory $\nu - e$ scattering experiments from the observed lack of distortions of the recoil electron energy spectra. Upper bounds on the neutrino magnetic moment have been obtained in recent reactor experiments: $\mu_\nu \leq 9.0 \times 10^{-11} \mu_B$ (MUNU collaboration [16]), $\mu_\nu \leq 7.4 \times 10^{-11} \mu_B$ (TEXONO collaboration [17]). The best world limit $\mu_\nu \leq 3.2 \times 10^{-11} \mu_B$ has been recently obtained by the GEMMA collaboration [18]. A stringent limit has also been obtained in the Borexino solar neutrino scattering experiments: $\mu_\nu \leq 5.4 \times 10^{-11} \mu_B$ [19]. Note that the magnetic and electric transition moments can contribute to the effective value of μ_ν (see Section 3.6 of [1]).

Neutrino electromagnetic interaction effects. If a neutrino has the non-trivial electromagnetic properties discussed above, a direct neutrino coupling to photons is possible and several processes important for applications exist [12]. A set of most important neutrino electromagnetic processes is: 1) neutrino radiative decay $\nu_1 \rightarrow \nu_2 + \gamma$, neutrino Cherenkov radiation in an external environment (plasma and/or electromagnetic fields), spin light of neutrino, $SL\nu$, in the presence of a medium [20]; 2) photon (plasmon) decay to a neutrino-antineutrino pair in plasma $\gamma \rightarrow \nu\bar{\nu}$; 3) neutrino scattering off electrons (or nuclei); 4) neutrino spin (spin-flavor) precession in a magnetic field (see [21]) and resonant neutrino spin-flavour oscillations in matter [22]. The tightest astrophysical bound on neutrino magnetic moments, $(\sum_{i,j} |\mu_{ij}|^2)^{1/2} \leq 3 \times 10^{-12} \mu_B$, applicable to both Dirac and Majorana neutrinos, has been obtained from the observed lack of anomalous stellar cooling due to plasmon decay [12].

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- [1] Giunti C and Studenikin A 2009 *Phys.Atom.Nucl.* **72** 2151, arXiv: 0812.3646
- [2] Kayser B 1982 *Phys.Rev. D* **26** 1662
- [3] Nieves J F 1982 *Phys.Rev. D* **26** 3152
- [4] Nowakowski M, Paschos E and Rodriguez J 2005 *Eur.J.Phys.* **26** 545
- [5] Marciano W J and Sanda A I 1977 *Phys.Lett. B* **67** 303
- Lee B W and Shrock R E 1977 *Phys.Rev. D* **16** 1444
- Fujikawa K and Shrock R E 1980 *Phys. Rev. Lett.* **45** 963
- [6] Schechter J and Valle J W F 1981 *Phys.Rev. D* **24** 1883
- [7] Kobzarev I and Okun L 1972 *Problems of Theoretical Physics* (Moscow: Nauka) p 219
- [8] Foot R, Lew H and Volkas R R 1993 *J. Phys. G* **19** 361; *ibid.* 1067 [Erratum]
- Babu K S and Mohapatra R N 1989 *Phys.Rev. D* **63** 938
- [9] Bardeen W, Gastmans R and Lautrup B 1972 *Nucl.Phys. B* **46** 319
- Cabral-Rosetti L, Bernabeu J, Vidal J and Zepeda A 2000 *Eur.Phys.J. C* **12** 633
- [10] Dvornikov M and Studenikin A 2004 *Phys.Rev. D* **69** 073001
- [11] Marinelli M and Morpurgo G 1984 *Phys.Lett. B* **137** 439
- [12] Raffelt G 1996 *Stars as Laboratories for Fundamental Physics* (Univ. of Chicago Press)
- [13] Bernabeu J, Papavassiliou J and Vidal J 2004 *Nucl.Phys. B* **680** 450
- [14] Fujikawa K and Shrock R 2004 *Phys.Rev. D* **69** 013007
- Bernabeu J, Papavassiliou J and Binosi D 2005 *Nucl. Phys. B* **716** 352
- [15] Pal P and Wolfenstein L 1982 *Phys.Rev. D* **25** 766
- [16] MUNU Collab. (Darakchieva Z *et al.*) 2005 *Phys.Lett. B* **615** 153
- [17] TEXONO Collab. (Wong H T *et al.*) 2007 *Phys.Rev. D* **75** 012001
- [18] Beda A G *et al.* 2009 *Particle Physics on the Eve of LHC* ed. by Studenikin A (World Scientific: Singapore) p 112, arXiv:09.06.1926.
- [19] Borexino Collab. (Arpesella C *et al.*) 2008 *Phys.Rev.Lett.* **101** 091302
- [20] Lobanov A and Studenikin A 2003 *Phys.Lett. B* **564** 27; *ibid.* 2004 **601** 171
- Grigoriev A, Studenikin A and Ternov A 2005 *Phys.Lett. B* **622** 199
- Studenikin A 2008 *J.Phys.A: Math.Theor.* **41** 164047
- [21] Okun L, Voloshin M and Vysotsky M 1986 *Sov.Phys.JETP* **64**, 446
- [22] Lim C and Marciano W 1988 *Phys.Rev. D* **37** 1368
- Akhmedov E 1988 *Phys.Lett. B* **213** 64